A fundamental principle in economics is the characterization of future states of nature—both in terms of the time (which is deterministic) and the well-being of the economy. For example, a bottle of water is more valuable in Miami on September 22, 2022, if there had just been a major hurricane, and the streets are flooded, etc., than if the weather is nice, and everything is normal. $1,000 is worth more on November 16, 2030 if the economy is in a deep recession and the stock market has crashed, than if on that date the economy is booming and the stock market is at record highs.

When we study the time-value of money and the pricing of US Treasury securities, we focus only on the deterministic passage of time. In general receiving $1 today is worth more to us than a commitment to receive $1 in 6 months. The reason is that we are impatient. The cost of impatience—that is the interest rate—has varied greatly over time. In the business context money today can be invested. So business demand for financial capital (money) depends on the productivity of physical capital, as well as businesses' expectations about the future well-being of the economy.

A final ingredient in the cost of money is the fact that the supply of money itself is not known, and for the most part we consider nominal interest rates. So when high inflation is expected, nominal interest rates have to offset the expected devaluation of the currency.

The period from mid-2007 through 2017 experienced historically low interest rates. This is because the financial crisis resulted in severe pessimism about the future well-being of the economy and inflationary expectations were very low.

We can measure the time value of money in the economy by looking at US Treasury securities. The US Treasury is considered to be free of default risk. The Treasury issues marketable securities using highly-developed, regular auctions. Because these securities are available in large supply and are traded in a liquid market, we can use them as benchmarks for the cost of money.

The Treasury issues the following marketable securities:

- Nominal securities:
    - Treasury Bonds
    - Treasury Notes
    - Treasury Bills
- Real securities:
    - Treasury Inflation-Protected Securities (TIPS)

A US Treasury bill is a debt obligation from the United States Treasury. Currently the Treasury sells one-year bills using a sealed-bid auction, once a month. It also sells 6-month bills every week. A bill is a security that has a term to maturity that is 1-year or less. It sells at a discount to its par value and makes no interest payments.

Consider that you buy a one-year bill on October 5, 2017 for 97.75. This bill matures on October 4, 2018. The annually compounded rate of return –which is also called the yield-to-maturity, is the interest rate that the owner will make if she holds the bill to maturity. In this case: 97.75 = 100/[(1+y)^T]. T is the number of years in the security's term. Another way of writing this is: 97.75(1+y)^T = 100. In this case T=1, so: 1+y = 1.023, so y = 2.3%.

This is useful information, as it tells us the pure time-value of money in our economy. This reflects a lot of things, including the rate of return on investments in the economy, the rate of impatience, and the expected rate of inflation. This simple exercise highlights one of the major themes in finance. Financial markets contain information about the costs of time and risk, which financial managers can use to make decisions.

Because a bill does not make interest payments we also call it a zero-coupon bond. The term *bill* is used for non-interest paying securities with original maturity of 1 year or less.

As their name suggests, STRIPS are another kind of zero-coupon bond stripped away from US Treasury notes and bills (using the Commercial Book-Entry System). The Treasury does not issue STRIPS directly, but allows institutions to break-down their notes and bonds into each of the component payments, which are then traded separately. If you buy a STRIPS that matures on May 15, 2027 on November 15, 2017, for 75% of par, then your annually compounded yield-to-maturity is 3.04%. T is the number of years in the life of the 0-coupon bond. In this case, T = 9.5.

$$75(1+y)^T = 100$$

$$(1+y)^T = 1.333333$$

$$1+y = 1.333333^{(1/T)}$$

$$1+y = 1.0304$$

Compounding means that the interest is added to the principal, so that it also earns interest. Many compounding conventions use the timing of the cash flows as the frequency of compounding. The STRIPS makes no intermediate cash flows, so the compounding period is important for comparisons.

When we analyze the present value of future cash flows it is a good idea to draw a time-line that shows all cash flows on all dates. For a zero-coupon bond, this is quote simple.

Effective annual yield and other compounding intervals:

Suppose that Citibank offers a 2-year CD at an annually compounded rate of 3% and BofA offers a 2-year CD at a continuously compounded rate of 2.9%. Which is better?

The value of $100 at the end of 1 year if it earns 5% compounded:

Annually: 100(1.05) = 105

Semi-annually: 100(1 + .05/2)^2 = 105.0625

Quarterly: 100(1 + .05/4)^4 = 105.094534

Monthly: 100(1 + .05/12)^12 = 105.11619

Daily: 100(1 + .05/365)^365 = 105.12675

Notice that the interest earned in each compounding period gets smaller as the number of compounding period increases.

Continuous: 100 e^.05 = 105.12711

We use the construct of effective annual yield to put interest rates with various compounding intervals. This is the answer to the question of what rate if compounded annually would give us the same rate of return at the end of a year.

Since we used $100 as the base of our CD above, we can see the EAY by inspection: for example, the effective annual yield from 5% compounded monthly is 5.11619%.

So now we can answer the posed question.

Effective annual yield from CD on maturity from Citibank: 3%
Effective annual yield from CD on maturity from BofA: 2.942459

So the Citibank CD is a better deal.

Refer back to our STRIPS that has a 3.04% yield-to-maturity. The continuously compounded yield-to-maturity can be directly computed:

$75 \exp(yT) = 100$
$yT = \ln(100/75)$
$yT = .28768207$

T is the number of years until the STRIPS matures = 9.5, so:
y = 3.028%. Note that this continuously compounded yield is less than the annually compounded yield. You should be able to explain why that is so.

It should be clear that the time value of money allows you to go both forward through time – what will $100 placed in a 2-year CD that earns 2% compounded semi-annually be worth after two years; and backward through time – what is the value of a 2-year STRIPS is the two-year semi-annually compounded spot rate is 2%.

In general, when you analyze cash flows over time it is a good idea to write a time-line that shows the time that each cash flow occurs.

Notes and bonds are debt obligations that typically make semi-annual interest payments. Consider, the 2.75% May 15, 2027 Treasury note. This is an IOU from the US Treasury that will pay par on maturity plus 1.375% of par on every May 15 and November 15 from now until maturity. We use 100 as par—since bond prices are quoted in terms of percentage of par.

Because of this convention, we use the term bond equivalent yield to refer to interest compounded semi-annually.

We want to think of a coupon bond (or note) as a portfolio of zero-coupon bonds. That is, we envision each interest payment as a 0-coupon bond. The reason for this is that each of these separate payments (or 0-coupon bonds) occurs at a different time and must be discounted at the appropriate discount for that time.

Our objective now is to learn about these discount factors from market information. To this end we will no bootstrap the yield curve. In the process we also apply the law of one price (which is the same as the absence of arbitrage opportunities) in financial markets.

The first step in this exercise is to collect bond prices such that we have one bond that matures at the end of the next 6-month periods. We use the bond that matures in 6 months to obtain the six-month spot rate. Even though the terminal payment on this security includes both interest and principal it is effectively a 0-coupon bond from today's perspective because there is only 1 future cash flow.

Armed with the 6-month spot rate we can isolate the value of the interest payment in 6 months on the 1-year bond. If we subtract this value from the bond's price we isolate the market's value of the cash flow that occur in 1 year, and we can therefore determine the 1-year spot rate.

Notice that we have to proceed sequentially so that we have only 1 spot rate to determine from each note. Also observe that we are invoking the law of one price by the fact that $1 received in 6 months has the same value—regardless of whether it comes from an interest payment on a 6-year bond or a principal payment on a 6-month bond.