Option Empirics I.

Christopher G. Lamoureux

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Breeden and Litzenberger 1978

Consider three call options on S that expire in 3 months, with strike prices of \$49.99, \$50, and \$50.01. a butterfly spread (B) that is long the outside 2 calls and sells 2 of the middle call.

If the price of S in 3 months is \$49.99 or lower, all three calls expire worthless. If the price is \$50, you make 1 cent. If the price is \$50.01 or higher, you make 0.

So this butterfly spread portfolio (actually $100 \cdot B$) is an Arrow-Debreu security. Its price tells us something about the EMM distribution of S. In particular, $e^{-rT}100 \cdot B$ is the probability that S will be \$50 in $T = 3$ months. So, if we had a continuum of options with strikes ranging from 0 to ∞, that expire on T, we could trace out the EMM density of S on T.

Breeden and Litzenberger (Continued)

Since we can write the limit of the second derivative of the call price, C, wrt the strike price, X:

$$\frac{\partial^2 C}{\partial X^2} = \lim_{h \to \infty} \frac{C(X+h) - 2C(X) + C(X-h)}{h^2}$$

We note that our butterfly portfolio converges in the limit to the numerator of this expression.

Lamoureux and Lastrapes 93. 1

Two perspectives on the paper:

1. A volatility forecasting horse race.
2. An orthogonality restriction test of an asset pricing model.

Model: Hull and White (1987), where the variance follows a geometric Brownian motion.
Data: 10 individual stocks. April 19, 1982 – March 30, 1984. (Pre Oct 19, 1987 crash). ATM options, 90 - 180 day terms. All inside quote pairs within each day used to get one IV per day.

Lamoureux and Lastrapes 93. 2

Step 1: Show the size of the bias in the implied volatility resulting from Jensen's Inequality.

1. Simulate return and variance under the model (with ρ estimated from data). (Discrete simulation using Box-Müller discretization.)
2. Option price is computed by Monte Carlo integration.
3. Implied volatility from BS assumption is obtained from Option price.
4. This is compared to the mean of the (simulated) variance process.

In all cases the bias arising from Jensen's Inequality is less than 1% of the variance.

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Step 2:
Include the implied variance in the GARCH specification–both with and without the GARCH parameters.
When the GARCH terms are not included, the coefficient on the implied volatility averages 1.2.
When GARCH terms areincluded, the coefficient on the implied volatility generally drops and loses statistical significance.
Issues with this test: Temporal alignment.

Lamoureux and Lastrapes 93. 4

Step 3:
Evaluate RMSE of alternative forecasts of the path of volatility. Compare to sample variance. 4 alternative models:

1. Implied variance.
2. Updated GARCH.
3. Rolling GARCH.
4. Historical variance.

Results:

1. Updated GARCH always beats Rolling GARCH.
2. Historical vol tends to beat GARCH.
3. For 9 of 10 companies, GARCH beats IV.

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Step 4:
Out-of-sample "encompassing regression." Horse race.
Regress realized variance over the remaining life of the option on:

1. Updated GARCH forecast.
2. Historical variance.
3. Implied vol.

$t-$statistics non-standard.
Results:

1. Intercept is positive and significant.
2. IV is usually positive and significant–mean coef. 0.47.
3. Coefficient on GARCH statistically insignificant.
4. Coefficient on historical vol negative and usually significant.

Lamoureux and Lastrapes 93. 6

Interpretations:

1. GARCH is not good at long horizons (beyond one month). (Lamoureux and Lastrapes *JBES* 1990).
2. RMSE comparisons can be misleading (Fair & Shiller 1990).
3. Option prices contain useful information about future variances.
4. Do the results imply that option prices over-react or under-react to volatility shocks?

Buraschi and Jackwerth 2001. 1.

Buraschi and Jackwerth (*RFS* 2001) is a neat paper that examines the martingale restriction under Black and Scholes. The Fundamental Theorem of Finance applied to the Black and Scholes world implies that there exists a unique martingale measure — here there is a process: ξ with the properties that $\xi_0 = 1$ and $\xi_t S_t$ is a martingale.

$$\xi_t S_t = E[\xi_T S_T | F_t] \;\forall\; t \leq T$$

Recall that under Black and Scholes:

$$S_t = S_0 e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\omega_t}$$

and

$$B_t = B_0 e^{rt}$$

Buraschi and Jackwerth 2001. 2.

We need the following result:
If $X \sim \mathcal{N}(\mu, \sigma^2)$ then:

$$E[e^{\alpha X}] = e^{\alpha\mu + \frac{1}{2}\alpha^2\sigma^2}$$

Now, since ω_t is standard Brownian motion, it is $\sim \mathcal{N}(0, t)$.
So:

$$\begin{aligned} E[S_t] &= S_0 e^{(\mu - \frac{1}{2}\sigma^2)t} E[e^{\sigma\omega_t}] = \\ &= S_0 e^{\mu t} \end{aligned}$$

Further, let $\lambda = (\mu - r)/\sigma$ or $\mu = r + \sigma\lambda$.
Then:

$$E[S_t] = S_0 e^{rt + \lambda\sigma t}$$

Buraschi and Jackwerth 2001. 3

So now if we let $\xi_t = e^{-\left(r+\frac{1}{2}\lambda^2\right)t-\lambda\omega_t}$,
then $E[\xi_t S_t]$ is a martingale.

Background

Rubinstein (1976 *Bell Jrnl*) notes that option pricing is (of course) a special case of the Fundamental Theorem of Finance. That is, in the absence of arbitrage:

$$C_t(S, n, K) = E_t\left[M_{t,t+n} \cdot (S_{t+n} - K)^+\right]$$

M is a positive random variable.
If we assume that S and M are conditionally bivariate log-normal then this results in the Black-Scholes formula.
Now the link between M and ξ:

$$M_{t,t+n} = \frac{\xi_{t,t+n}}{\xi_t}$$

And BJ show:

$$\ln M_{t,t+n} = \frac{1}{2}\frac{\mu}{r\sigma^2}\left(\mu - \sigma^2\right)\ln\left(\frac{Bt, t+n}{B_t}\right) + \frac{-\mu}{\sigma^2}\ln\frac{S_{t,t+n}}{S_t}$$

Empirics

BJ note that in the original Black-Scholes model, M is a constant weight function (they call β) times the return on the bond and stock.
They extend the analysis to the case where the volatility is a time-varying deterministic function of the stock's price. In this setting the bond and stock span the state space, but β varies through time.
Data: S&P 500 Options April 2, 1986 - December 29, 1995.
Idea: Return on the Index (implied from futures price) and ATM call are sufficient to characterize M.

Coval and Shumway 1.

Start by noting that option returns have 2 components:

1. Leverage: Returns on calls on stocks with positive risk premia should:
    1.1 be positive, and
    1.2 increase in the strike price.
2. Convexity: Net of leverage, options should earn no risk premium under the Black-Scholes assumptions (i.e., options are redundant).

Coval and Shumway 2.

Note that under CAPM and Black-Scholes, a call's β is:

$$\beta_C = \Delta \frac{S}{C} \beta_S$$

Data: S&P 500 Options January 1990 – October 1995.