Option Empirics II.

Christopher G. Lamoureux

February 6, 2013

Coval & Shumway

CS evaluate the weekly returns on a delta-neutral position using a put and call (i.e., straddle). The returns are $< -150\%$ per year, on the SP500.
From CS (p. 998)

> *These results strongly suggest that some risk other than market beta risk is being priced in option returns. Investment strategies that exploit the apparent overpricing of calls and puts need not assume high degrees of market risk. By simultaneously shorting both options – by selling straddles – investors can earn excess returns of around 3market risk.*

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CS also find that these straddle returns are highest when the call is deep in-the-money, (and therefore the put is deep out-of-the money).
In any sample, one might argue that this is due to a Peso problem.
CS pursue this possibility by also looking at the returns of "crash-neutral straddles." –Purchase a straddle and sell a deep out-of-the money put.
They still report an average annual return of $> 150\%$.

Branger & Schlag

BS note that the spanning test of Buraschi and Jackwerth (e.g.), is essentially the same as the statement that the return on a hedge portfolio must: a) on average vanish; and b) be uncorrelated with information available at t.
BS consider:

$$dS = (\mu - \mu_X h^P)Sdt + \sqrt{V}Sd\omega^S + S(e^X - 1)dN$$
$$dV = \kappa(\theta - V)dt + \sigma_V\sqrt{V}\left(\rho d\omega^S + \sqrt{1-\rho^2}d\omega^V\right)$$

They follow Heston (and CIR) and assume that the risk premium on volatility is linear in volatility:

Branger & Schlag 2.

This means that the process in the equivalent martingale measure is:

$$dS = (r - \mu_X h^Q) S dt + \sqrt{V} S d\omega^S + S(e^X - 1) dN$$
$$dV = \kappa(\theta - V) dt - \lambda \sigma_V \sqrt{V} \left(\rho d\omega^S + \sqrt{1 - \rho^2} d\omega^V \right)$$

So, from Itô's Lemma:

$$\begin{aligned} dC &= rCdt + \frac{\partial c}{\partial S}(dS - rSdt) + \frac{\partial c}{\partial V}\{dV - \kappa(\theta - V)dt + \sigma_V \lambda V dt\} \\ &+ f(t, S, V, X)dN - E^Q[f(t, S, V, X)|\mathcal{F}]\, h^Q dt \end{aligned}$$

f is the jump risk:

$$f(u, S, V, X) = c(u, Se^X, V) - c(u, S, V) - \frac{\partial c}{\partial S}(u, S, V) S(e^X - 1)$$

Branger & Schlag 3.

BS note that the expected excess return on the call over the next instant is:

$$\frac{1}{C}\left\{\frac{\partial c}{\partial S}(\mu - r)S + \frac{\partial c}{\partial V}\sigma_V \lambda V + E^Q\left[f(t,S,V,X)|\mathcal{F}\right]\left(h^P - h^Q\right)\right\}$$

So, they write the expected hedging error:

$$\begin{aligned} & IE^S(\mu - r) + IE^V\lambda + IE^J(h^P - h^Q) \\ + \quad & (\mu - r)\left\{DE^S(\mu - r) + DE^V\lambda + DE^J(h^P - h^Q)\right\} \\ + \quad & (\mu - r)ME \end{aligned}$$

The first row (IE terms) is the expected hedging error under ideal conditions (continuous rebalancing and correct model); the second row is the expected hedging error with discrete rebalancing; while the third row is the expected hedging error that results from using the wrong model.

Branger & Schlag 4

Look at each of the hedging terms under ideal conditions:

$$IE^S(t, t+\tau) = \int_t^{t+\tau} e^{r(t+\tau-u)} E^P\left[\left(\frac{\partial c}{\partial S}(u, S, V) - H\right) S \,\middle|\, \mathcal{F}\right] du$$

$$IE^V((t, t+\tau) = \int_t^{t+\tau} e^{r(t+\tau-u)} E^P\left[\left(\frac{\partial c}{\partial V}(u, S, V)\sigma_V V\right) \,\middle|\, \mathcal{F}\right] du$$

$$IE^J((t, t+\tau) = \int_t^{t+\tau} e^{r(t+\tau-u)} E^P\left[(f(u, S, V, X)\sigma_V V) \,|\, \mathcal{F}\right] du$$

In the general SVJ model, the expected hedging error is negative if:

1. MPVR is negative, and/or
2. $h^Q > h^P$.

IE^V increases in option's time to maturity whereas IE^J decreases in option's time to maturity.

Branger & Schlag 5. Discretization Errors

If delta hedge is implemented in discrete time, the the expected hedging error between resets is:

$$
\begin{aligned}
&IE^V(t_i, t_{i+1})\lambda + IE^J(t_i, t_{i+1})(h^P - h^Q) \\
&+ (\mu - r)\Big[DE^S(t_i, t_{i+1})(\mu - r) + DE^V(t_i, t_{i+1})\lambda \\
&\qquad + DE^J(t_i, t_{i+1})(h^P - h^Q)\Big]
\end{aligned}
$$

$$
DE^S(t_i, t_{i+1}) = \int_{t_i}^{t_{i+1}} e^{r(t_{i+1} - t_i)} E^P \left[S_t \int_{t_i}^{t} \frac{\partial^2 c}{\partial S^2}(u, S_u, V_u) S_u du \middle| \mathcal{F}_t \right] dt
$$

Branger & Schlag 6. Discretization Errors

$$DE^V(t_i, t_{i+1}) = \int_{t_i}^{t_{i+1}} e^{r(t_{i+1}-t_i)} E^P \left[S_t \int_{t_i}^{t} \frac{\partial^2 c}{\partial S \partial V}(u, S_u, V_u) \sigma_V V_u du \middle| \mathcal{F}_t \right] dt$$

$$DE^J(t_i, t_{i+1}) = \int_{t_i}^{t_{i+1}} e^{r(t_{i+1}-t_i)} E^P \left[S_t \int_{t_i}^{t} E^Q[E^Q[g(u, S_u, V_u, X_t)|\mathcal{F}_u]du \middle| \mathcal{F}_t \right] dt$$

$$g(t, S, V, X) = \left[\frac{\partial c}{\partial S}(t, Se^X, V) - \frac{\partial c}{\partial S}(t, S, V) \right] e^X - \frac{\partial^2 c}{\partial S^2}(u, S, V) S(e^X - 1)$$